Cracking-Induced Mistuning in Bladed Disks

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ABSTRACT

A study of the mechanisms of cracking-induced mistuning in bladed disks is presented in this paper. An analytical model for a bladed disk was formulated using lumped-mass-beams, and a cracked blade was represented by a beam with a through-crack at the root. Local stiffness reduction due to cracking was incorporated using a flexibility matrix method. The dynamic characteristics of the bladed disk were analysed for the tuned system and then for the mistuned system with cracks of various depths introduced at the root of a blade. The dynamic characteristics of the mistuned system with a cracked blade were evaluated and the mistuning pattern due to blade cracking was investigated. The mechanisms of cracking-induced mistuning were elucidated by analysing the mode localisations and the reductions in natural frequency for different coupling ratios between the disk and blades. The results show that the occurrence of a cracked blade significantly increases the response amplitude of the other uncracked blades and therefore gives rise to enhanced HCF damage for all coupling ratios studied. In the present study, the relationship between the cracked blade and its vibration response signature is established, hence providing the essential guidance required for blade crack detection procedures and for blade failure investigations.

1.0 INTRODUCTION

Bladed disks in a gas turbine generally exhibit cyclic symmetry with a cluster of closely spaced and repeated vibration modes. This type of eigenstructure may often be sensitive to small perturbations of blade geometry and material properties, which is inevitable due to manufacturing tolerances or to property deterioration of components during service. These small perturbations, known as mistuning, usually increase the maximum blade force response and consequently cause high vibratory stresses that may have a detrimental impact on the high cycle fatigue (HCF) life.

During the past two decades, research efforts in characterising mistuning and understanding the associated negative impact have been dramatically enhanced due to an increasing trend for high cycle fatigue failures caused by blade vibrations [1]. Adding to this concern is the increased use in modern engines of integrated bladed disks, or blisks, which have dynamic characteristics that add to their susceptibility to mistuning and associated HCF problems. An important aspect to this is the possible impact of mistuning that can be induced by blade cracking. It is generally recognised that blade mistuning is caused by small differences in individual blade properties, resulting in a split of vibration modes of the tuned system, and the sensitivity of dynamic responses to blade mistuning depends primarily upon the ratio of mistuning strength to coupling strength [2~3]. Under both cyclic and sustained mechanical and thermal loading, cracks may initiate and propagate in blades due to fatigue damage [4~6], resulting in local stiffness discontinuities that may alter the dynamic characteristics of bladed disks. The effects of a crack on the blade-alone vibration characteristics have been studied intensively by a large number of researchers [7~10], focusing on how to predict the lowered frequency due to

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cracking in a blade. However, study of cracking induced mistuning for bladed disks is very limited. The first attempt to study cracking induced mistuning was made by Kuang and Huang [11] with the development of an analytical model of a cracked bladed disk with shroud. Their study revealed that the mode localisation and associated mistuning could be introduced by a cracked blade. Hou [12] numerically demonstrated cracking induced mistuning for a weakly coupled research blisk using 3D finite methods. It was found that the natural frequencies of the cracked blade decreased significantly only when the crack was sufficiently large. However, the cracked blade dramatically changed the dynamic response of the blisk even when a small crack was present. As the study was applied to only a weakly coupled system, it is questionable whether this observation is generic enough to be applicable to systems with different coupling ratios because of the sensitivity of mistuning to the coupling ratio.

This paper presents a detailed study of cracking induced mistuning and the associated mode localisation under different coupling ratios, with the aim of identifying the possible relations between dynamic characteristics and crack parameters, and the possible impact of a cracked blade on other blades. An analytical model for a bladed disk was formulated using lumped masses and mass-less beams. The local stiffness reduction due to cracking was incorporated into the cracked blade using a flexibility matrix. The dynamic characteristics of the bladed disk were analysed for both the tuned system and the system with cracks of various depths introduced at the root of a blade. The mechanism of cracking-induced mistuning was elucidated by analysing the mode localisations and the reductions in natural frequency under different coupling ratios between the disk and blades. The results show that the occurrence of a cracked blade significantly increases the response amplitude of other uncracked blades and therefore gives rise to enhanced HCF damage for all coupling ratios studied. Also, the relationship between the cracked blade and its vibration response signature is established, hence providing the essential guidance required for blade crack detection procedures and for vibration based prognostic analyses.

2.0 FORMULATION OF THE BLADED DISK WITH A CRACKED BLADE

A lumped-mass-spring model grounded using a massless beam was used as a baseline of a bladed disk without a crack as shown Figure 1.

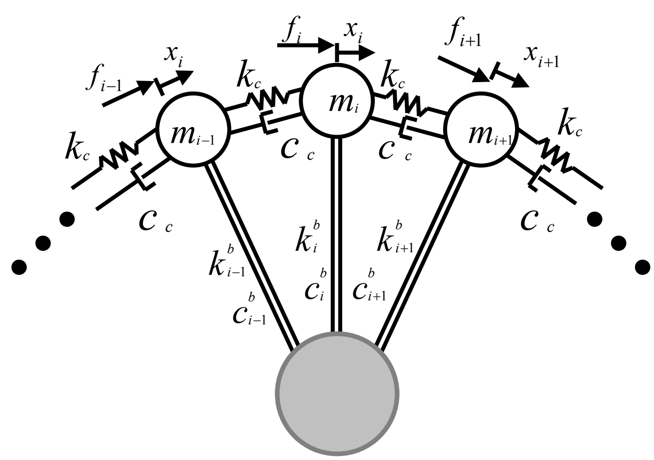


Figure 1. A lumped-mass-spring model of a bladed disk

This model has been used to study various aspects of mistuning in the literature and shown to be effective in capturing the essential dynamic characteristics of a mistuned bladed disk [2~3]. The equation of motion for the i^{th} mass representing the i^{th} blade ($i = 1,....,N$) can be described as:

$$m_i\ddot{x} + (c_i^b + 2c_c)\dot{x}_i - c_c(\dot{x}_{i-1} + \dot{x}_{i+1}) + (k_i^b + 2k_c)x_i - k_c(x_{i-1} + x_{i+1}) = f_i(t) \qquad (1)$$

Where $x_0 \equiv x_N$ to describe a closed system. Suppose that only the blade stiffness is mistuned as is the case when a crack is present, and define $k_i^b = k_b(1+\delta_i)$, $m_i = m_b$ and $c_i^b = c_b$. Assuming the coupling damping term $c_c = 0$, then the Equation (1) can be simplified to:

$$\ddot{x}_i + c_b / m_b\dot{x}_i + \omega_b^2(1+\delta_i + 2R^2)x_i - \omega_c^2(x_{i-1} + x_{i+1}) = f_i(t)/m_b \qquad (2)$$

Where $R^2 = \omega_c^2 / \omega_b^2$, is defined as the coupling ratio. The stiffness mistuning term δ_i defined in Equation 2 can be calculated as below:

$$\delta_i = (\omega_n^i / \omega_n^0)^2 - 1 \qquad (3)$$

The engine order excitation force can be expressed as:

$$f_i(t) = F_0 e^{(\omega t + (i-1)\theta)j} \qquad (4)$$

The inter-blade phase angle θ is defined as:

$$\theta = 2\pi E / N \qquad (5)$$

E is defined as the engine order. Assembling the equations (2~5) in matrix form and assuming proportional damping, the displacement response amplitude for the i^{th} blade in equation (2) can be solved as:

$$x_i(\omega) = F_0 / m_b \sum_{k=1}^{N} \frac{\{\phi_k\}^T \Phi \phi_{i,k}}{\omega_{n,k}^2 - \omega^2 + 2j\xi\omega_{n,k}\omega} \qquad (6)$$

where Φ is the phase vector defined as: $\Phi = \{e^{\omega t},...e^{(\omega t+(i-1)\theta)j},...e^{\omega t+(N-1)\theta})^T$. Assigning $\delta_i = 0$ in equation (2), the displacement $x_i^0(\omega)$ corresponding to a tuned system can be obtained.

To solve Equation 6, the remaining problem is to determine the fundamental frequency of a cracked blade so that the mistuning term due to a cracked blade can be incorporated. The relationship between the fundamental frequency of a cracked blade and the crack parameters can be established by using a single cracked beam and a lumped mass at the free end. Figure 2 shows a schematic of a single blade model represented by a massless elastic cantilever beam with a lateral crack at the root and a lumped mass at the free end.

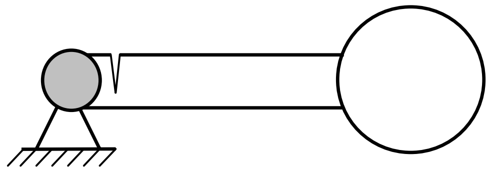


Figure 2. Schematic of a cracked massless elastic beam and mass

The local stiffness or flexibility matrix of the cracked beam under general loading conditions can be found in Irwin's work [13]. In the case of pure bending vibration, Dimarogonas [14] showed that the flexibility matrix could be simplified further to only bending related terms. Based on the fundamental frequency of an uncracked beam, the fundamental frequency of the cracked beam shown in Figure 2 can be expressed as:

$$\omega_n = \omega_n^0 / (1 + \mu^2) \tag{7}$$

where,

$$\mu^2 = 3EI / lK_t \tag{8}$$

$$K_t = E' b^3 h^3 / 6\Phi_f \tag{9}$$

$$\Phi_f = \int_0^a 2\tan\alpha \frac{0.923 + 0.199(1 - \sin(\alpha))^4}{\cos(\alpha)} d\alpha \tag{10}$$

$$\alpha = 2h / \pi a \tag{11}$$

Where b and h is the beam width and thickness respectively while a is the crack depth. Using Equations 7 to 12, the fundamental frequency of a cracked blade can be determined and the mistuning term defined in Equation 3 then can be calculated.

3.0 MODE LOCALISATION INDUCED BY A CRACKED BLADE

The effect of a cracked blade on mode localisation was investigated by considering different crack length ratios *a/W*, defined as the ratio between the crack length *a* and the blade root width *W*. a/W was in a range from 0.0 (without a crack) to 0.7 (the crack was 70% of blade width). The baseline model has the parameters of b=85 mm, h=5mm, l=170mm, m=0.37 kg. The relation between the fundamental frequency of a cracked blade and the crack length ratio was calculated using MatLab [15], as shown in Figure 3. The frequency ratio is defined as the value of the fundamental frequency of a cracked blade normalised by the blade frequency without a crack. As expected, the frequency of a cracked beam decreased monotonically with increasing crack length ratio due to the fact that a crack in a blade introduces only local flexibilities, which is consistent with the observations in the literature [7~11].

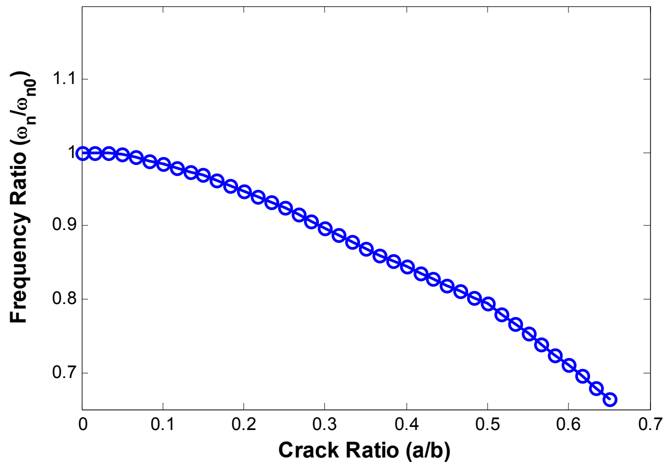
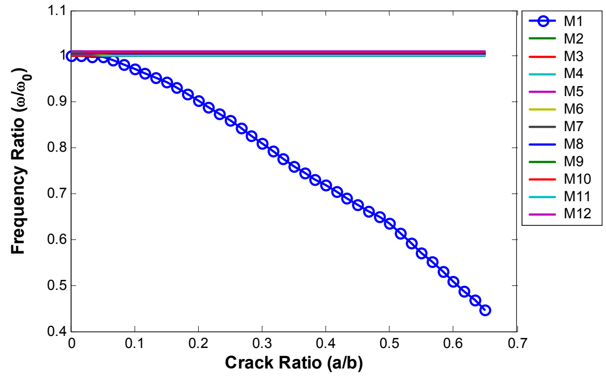


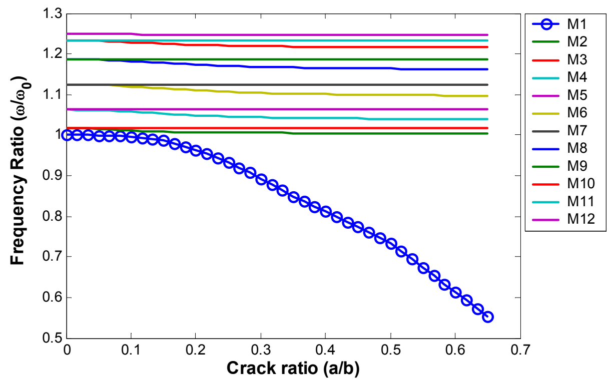
Figure 3. Relation between the fundamental frequency of a cracked blade and the crack length ratio

Figure 4 shows the relation between the natural frequencies of a 12 bladed disk and the crack length ratio. Three different disk-to-blade coupling ratios are considered, including weak coupling (R=0.05), moderate coupling (R=0.25) and strong coupling (R=1). The introduction of a crack mainly reduces the fundamental frequency of the bladed disk (one of a repeated pair) for both weakly and moderately coupled bladed disks. However, the reduction in the fundamental frequency is insignificant for a strongly coupled disk even if the crack length ratio is close to 0.7. For both weakly and moderately coupled bladed disks, the reduction in fundamental frequency is not noticeable when the crack length ratio is small, and only becomes significant when the crack length ratio is larger than 0.2. The fundamental natural frequency lowers dramatically after the crack length reaches 25% of the blade width. Note that the relation between a decrease in the fundamental frequency and the crack length ratio may not be considered as an eigenvalue loci veering phenomenon because of the monotonic decrease in the frequency with increase in the crack length ratio [2]. Therefore a system for the blade crack detection relying on monitoring relative changes in frequency may not be sufficient for the weakly or moderately coupled bladed disks until the crack is larger enough (larger than 25% of the blade width). In the case of strong coupling, the change in frequency cannot be used as an indicator for the presence of crack even if the crack is quite long (70% of blade width in this case).

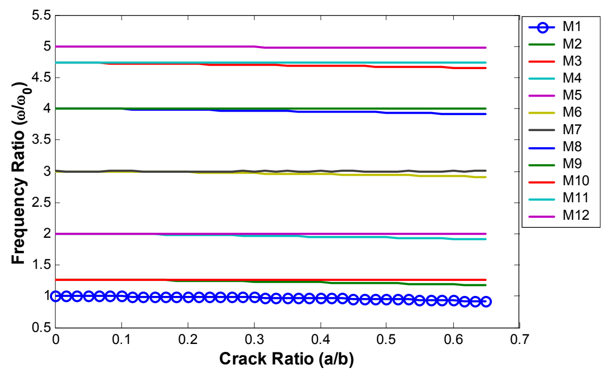
The relation between the normalised mode shapes (to 1 for each mode)) and the crack length ratio for the fundamental frequency is shown in Figure 5. The mode shape for the tuned case (based on a crack length ratio a/W=0) is a straight line corresponding to the zero nodal dimeter. This straight line is gradually distorted with increase in the crack length for all three coupling ratios until the peak displacements are finally localised to the cracked blade (Blade 1). Note that variation rates differ for the different coupling ratios and are more sensitive in weakly and moderately coupled systems. Hence mode localisation occurs for all three coupling ratios although a weakly coupled system is more sensitive than the other two systems.



(a) R=0.05

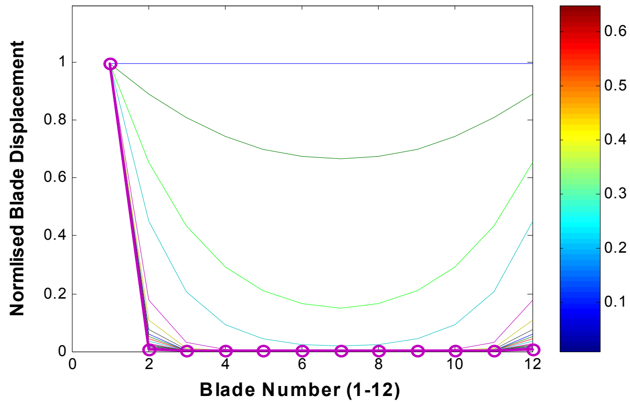


(b) R=0.25

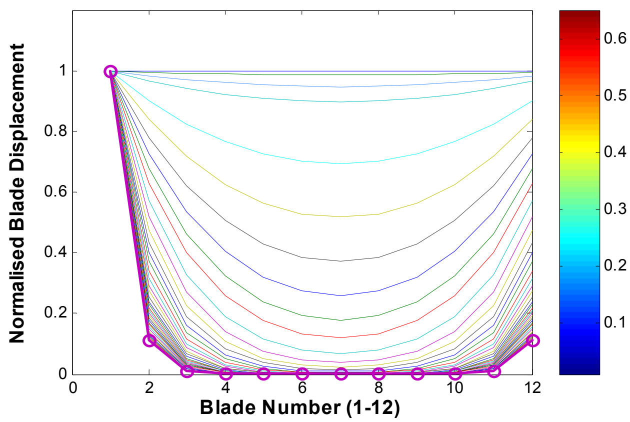


(c) R=1.0

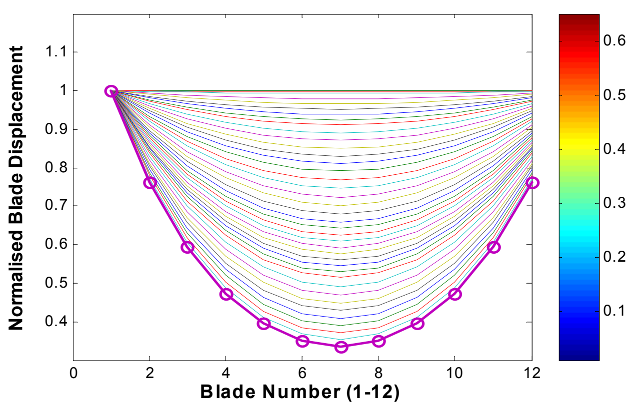
Figure 4. Variation of the 1st natural frequency with the crack length ratio



(a) R=0.05



(b) R=0.25



(c) R=1.0

Figure 5. Mode variation with crack length ratio

4.0 BLADE CRACKING-INDUCED MISTUNING

The effect of the cracked blade on the blade response amplitude was investigated by performing frequency response analyses of the bladed disks with different crack ratios. Figure 6 shows typical vibration response amplitudes of the blades for a weakly coupled system, normalised to the maximum tuned amplitude. Clearly, mistuning has been introduced by a cracked blade.

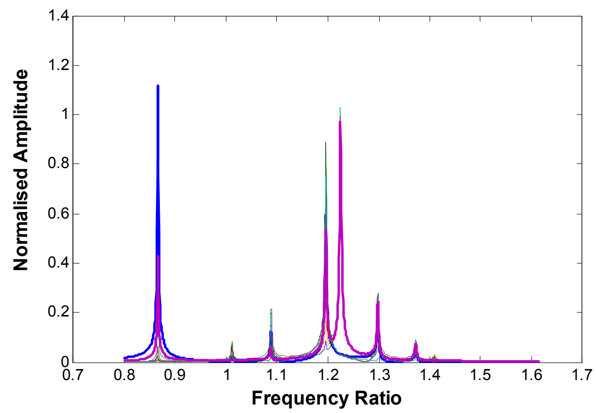
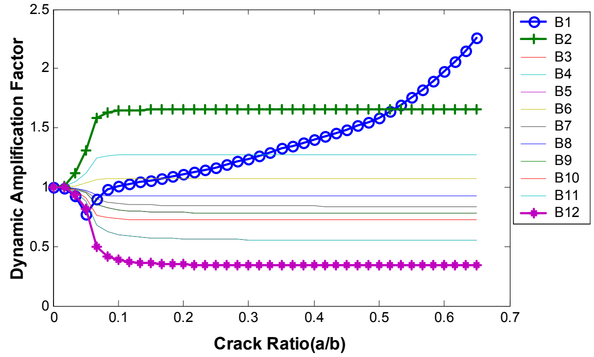


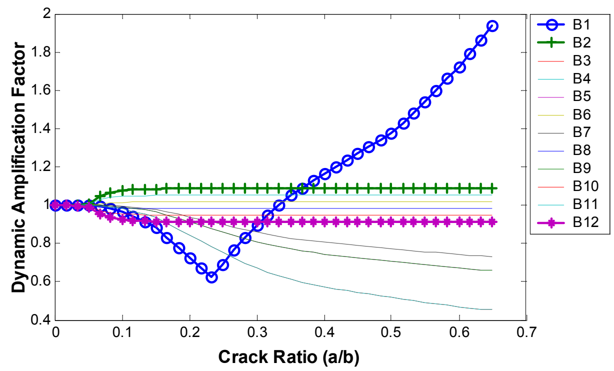
Figure 6. Frequency Response Amplitude of a bladed disk with a cracked blade (E=3, R=0.25, a/b=0.05)

Figure 7 shows the normalised maximum response amplitudes for all 12 blades in the bladed disk under three coupling ratios. It is found that the response amplitude of the cracked blade (Blade 1) is less than those of other blades under all the three coupling ratios when the crack length ratio is small. For a weakly coupled system, the vibration amplitude of the cracked blade becomes larger than the amplitude of the adjacent maximum responding Blade 2 only when the crack length ratio approaches to a limit value of 0.55. For the moderately coupled system, the limit value of the crack length ratio is 0.35. However, the limit value is greater than 0.6 for the strongly coupled system. Therefore, the cracked blade does not necessarily experience the maximum response in all scenarios, and the maximum responding blade varies depending on the crack length ratio. This may impose significant difficulties for vibration-based crack detection techniques if only the maximum responding amplitude is sought.

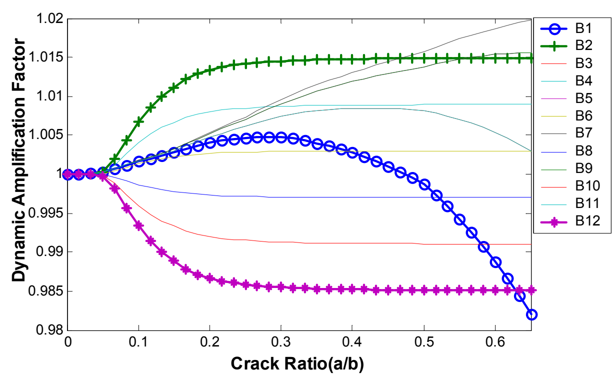
Nevertheless, the response variations of the two blades adjacent to the cracked blade show an interesting trend. Blade 2 (with a phase lag of $2\pi E/12$) experiences the maximum response amplitude for the crack length ratios approximately less than 0.5 for all three coupling ratios. Blade 12 (with a phase advance of $2\pi E/12$) experiences the minimum response amplitudes for ratios less than 0.5 for a weakly or strongly coupled system. However, this trend is not apparent for a moderately coupled system. This suggests that the cracked blade is only likely to be the maximum responding blade when a crack is sufficiently large. When the crack is small, the response amplitudes of adjacent blades are better indicators for detecting the existence of a cracked blade. This finding has an important implication for vibration-based blade crack detection and prognostic analysis as the cracked blade can be related to the response signatures of only a few blades in the whole bladed disk. Also the detection of the changes in vibration levels is an essential indicator for crack detection rather than the absolute response amplitude. As a cracked blade inevitably introduces mode localization and associated mistuning to a bladed disk, a cracked blade can cause higher response amplitudes to uncracked blades and consequently may have a detrimental effect on the high cycle fatigue life of blades.



R=0.05



(b) R=0.25



(c) R=0.50

Figure 7. Frequency Response Amplitude of Each Individual Blade (E=3)

5.0 CONCLUSION

The mistuning induced by a cracked blade in a bladed disk has been studied using an analytical model in order to understand the underlying mechanisms governing the blade dynamic response and the crack length. The following conclusions can be drawn from this study:

1. Mode localisation and associated blade mistuning can be introduced by a cracked blade in a bladed disk for all the three typical coupling ratios investigated, and the sensitivity of the mode localisation decreases with increasing coupling ratio.

2. For both weakly and moderately coupled systems, the effect of a cracked blade is to reduce the natural frequencies monotonically, but the frequency reduction is insignificant when the crack length is small. With increase in crack length, only the fundamental frequency shows a dramatic reduction and the reductions in other frequencies are negligible. For a strongly coupled system, the reduction in natural frequencies is insignificant regardless of the values of crack length.

3. A cracked blade does not necessarily experience the maximum response under all of the three typical coupling ratios, and the maximum responding blade depends on both the crack length and the coupling ratio. However, the mistuning caused by a cracked blade may amplify the dynamic response of the other uncracked blades and may give rise to HCF damage for other blades in a bladed disk.

4. As the maximum responding blade varies with the coupling ratio and crack length, a cracked blade cannot be easily identified based on response amplitudes when the crack is small. However, the adjacent blades do show a significant change in response amplitude. The adjacent blade with a phase lag may have an increased response whilst the blade with a phase advance may show a lowered response. This provides the essential information for vibration-based blade crack detection and prognostic analysis, as the cracked blade can be related to the response signature of uncracked blades.

It should be pointed out that a tuned system has been used as the baseline in this study and the only mistuning term is the local stiffness variation of a cracked blade. It is recommended that further study should include the effect from various random mistuning terms, i.e. variation of blade geometry and properties.

6.0 REFERENCES

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